

1) Evaluate the integral  $\iiint_E (xz - y^3) dV$ , where  $E = \{(x, y, z) \mid -1 \leq x \leq 1, 0 \leq y \leq 2, 0 \leq z \leq 1\}$ .

$$\boxed{-8}$$

2) Evaluate the iterated integral:

a)  $\int_0^1 \int_0^z \int_0^{x+z} 6xz \, dy \, dx \, dz$

b)  $\int_0^1 \int_0^z \int_0^y ze^{-y^2} \, dx \, dy \, dz$

a)  $\boxed{1}$

b)  $\boxed{\frac{1}{4e}}$

3) Evaluate the triple integral:

a)  $\iiint_E 2x \, dV$  where,  $E = \{(x, y, z) \mid 0 \leq y \leq 2, 0 \leq x \leq \sqrt{4 - y^2}, 0 \leq z \leq y\}$

b)  $\iiint_E 6xy \, dV$ , where  $E$  lies under the plane  $z = 1 + x + y$  and above the region in the  $xy$ -plane bounded by the curves  $y = \sqrt{x}$ ,  $y = 0$ ,  $x = 1$ .

c)  $\iiint_E xz \, dV$ , where  $E$  is the solid tetrahedron with vertices  $(0, 0, 0)$ ,  $(0, 1, 0)$ ,  $(1, 1, 0)$ , and  $(0, 1, 1)$ .

d)  $\iiint_E x^2 e^y \, dV$ , where  $E$  is bounded by the parabolic cylinder  $z = 1 - y^2$  and the planes  $z = 0$ ,  $x = 1$ ,  $x = -1$ .

e)  $\iiint_E x \, dV$ , where  $E$  is bounded by the paraboloid  $x = 4y^2 + 4z^2$  and the plane  $x = 4$ .

a)  $\boxed{4}$

b)  $\boxed{\frac{65}{28}}$

c)  $\boxed{\frac{1}{120}}$

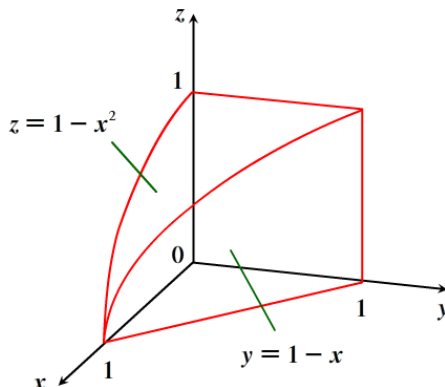
d)  $\boxed{\frac{8}{3e}}$

e)  $\boxed{\frac{16\pi}{3}}$

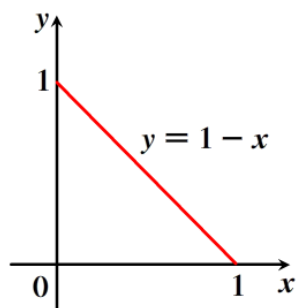
- 4) Use a triple integral to find the volume of the solid enclosed by the cylinder  $x^2 + y^2 = 9$  and the planes  $y + z = 5$  and  $z = 1$ . [Hint: use polar coordinates after integrating with respect to  $z$  ]

$$\boxed{36\pi}$$

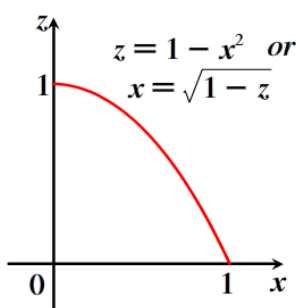
5) **Express** the iterated integral  $\iiint_E dV$ , where  $E$  is the solid drawn below, draw the projections of the solid onto each of the coordinate planes to aid you in expressing the following iterated integrals. **Do not solve the iterated integrals.**



xy-plane

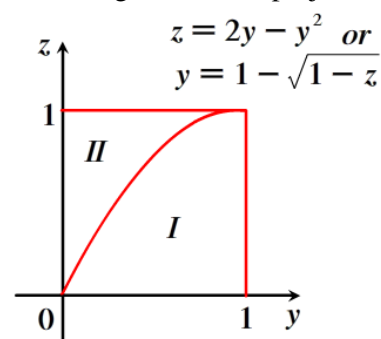


xz-plane



yz-plane

[Hint: 2 regions on this projection]



a)  $\iiint_E dy dx dz$   $\int_0^1 \int_0^{\sqrt{1-z}} \int_0^{1-x} dy dx dz$

b)  $\iiint_E dz dx dy$   $\int_0^1 \int_0^{1-y} \int_0^{1-x^2} dz dx dy$

c)  $\iiint_E dz dy dx$   $\int_0^1 \int_0^{1-x} \int_0^{1-x^2} dz dy dx$

d)  $\iiint_E dy dz dx$   $\int_0^1 \int_0^{1-x^2} \int_0^{1-x} dy dz dx$

e)  $\iiint_E dx dy dz$   $\int_0^1 \int_0^{1-\sqrt{1-z}} \int_0^{\sqrt{1-z}} dx dy dz + \int_0^1 \int_{1-\sqrt{1-z}}^1 \int_0^{1-y} dx dy dz$

f)  $\iiint_E dx dz dy$   $\int_0^1 \int_0^{2y-y^2} \int_0^{1-y} dx dz dy + \int_0^1 \int_{2y-y^2}^1 \int_0^{\sqrt{1-z}} dx dz dy$

- 6) Find the moments of inertia for a cube of constant density  $k$  and side length  $L$  if one vertex is located at the origin and three edges lie along the coordinate axes.

$$I_x = I_y = I_z = \frac{2}{3} kL^5$$

- 7) The joint density function for random variables  $X$ ,  $Y$ , and  $Z$  is:

$$f(x, y, z) = \begin{cases} Cxyz & \text{if } 0 \leq x \leq 2, 0 \leq y \leq 2, 0 \leq z \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- a) Find the value of the constant  $C$ .
- b) Find  $P(X \leq 1, Y \leq 1, Z \leq 1)$
- c) Find  $P(X + Y + Z \leq 1)$

a)  $C = \frac{1}{8}$

b)  $\frac{1}{64}$

c)  $\frac{1}{5760}$